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Abstract

This paper concludes that, when offenders are wealth constrained and the government is resource constrained and can commit to a certain policy throughout the whole planning horizon, cost minimizing deterrence is decreasing, rather than increasing, in the number of offenses. By extending the framework, suggested in Emons (2003), to n-periods setting, we prove that for the agents who may commit an act several times, optimal sanctions are such that the fine for the first crime equals the offender's entire wealth, and the fines are zero for all the subsequent crimes. This result contradicts the widely prevailing escalating penalties imbedded in many penal codes and sentencing guidelines. In addition, analogous to Emons (2004), this scheme does not appear to be a time consistent (subgame perfect) strategy for the government in an n-periods setting. Moreover, we show that, if the government cannot commit, equal rather than decreasing sanctions will be optimal.

JEL-Classification: D82; K41; K42

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1 Introduction

In this paper we concentrate on policies for crime control that are not only aimed at reducing the number of violations but are also cost minimizing from the point of view of

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the regulator. Unfortunately, these two objectives conflict with each other. Reduction of expenditures on crime control will lead to a lower deterrence rate and vice versa. However, both objectives seem to be very important for society. Society is better off when both the number of violations and the costs of crime control are reduced.

Another important question addressed in this paper is whether the optimal sanction scheme should be decreasing or increasing in the number of offenses. For the law and economics literature on optimal law enforcement, escalating sanction schemes, embedded in most sentencing guidelines, are still a puzzle. Garoupa (1997) or Polinsky and Shavell (2000) give excellent surveys of this literature.

The purpose of this study is to find the optimal penalty scheme which takes into account the two objectives, mentioned above. We study the problem of optimal sanctions for repeat offenders in a multi-periods model employing the two-periods framework suggested in Emons (2003). We assume that agents may commit a crime several times. The criminal act is inefficient, it causes harm for society; the agents are thus to be deterred. An important assumption of the model is that the agents are wealth constrained so that increasing the fine for the first offence means a reduction in the possible sanction for the subsequent offences and vice versa. A simplification compared to Emons (2003) is that in the forward looking solution we consider only history independent strategies of the agents. The government seeks to minimize the probability of apprehension and the number of crimes, since it is costly for society.

The main result is that the optimal penalty scheme is decreasing in the number of offenses. We find that it is optimal to set the sanction for the first detected offense equal to the entire wealth of the agent while the sanctions for all the subsequent offenses equal zero.

In this paper we discuss a general set up with representative offender and regulator whose aim is to block violations of law. However, it is clear that a similar framework can be applied in case of an antitrust authority dealing with a group of firms that form an illegal cartel. Antitrust law violations often are committed repeatedly by the same firm. Remarkably, sentencing guidelines in both US and Europe attach a higher gravity factor to recidivistic violations and, hence, prescribe to punish repeated offenders more heavily. Clearly, this does not go in line with the main results of the Emons (2003) work and our analysis. This puzzle still requires deep investigation in the law and economics literature. From the other point of view, our model, where offenders are wealth constrained, captures another important feature of current penalty schemes, namely, the existence of upper bound for the fine. Usually, this upper bound is given by either 10% of overall turnover of enterprise or by fixed monetary amount. The motivation for existence of this rule can be connected to the fact that antitrust authorities should not force firms to go bankrupt, in other words, the fact that firms are wealth constrained is taken into account.

We start the discussion with a review of the related literature. Rubinstein (1980) considers a setup where an agent can commit two crimes. A high penalty for the second crime is exogenously given. Rubinstein shows that for any set of parameters there exists a utility function such that deterrence is higher if the sanction for the first crime is lower than the sanction for the second crime. Landsberger and Meilijson (1982) develop a dynamic model with repeated offenses. They studied how prior offenses should affect the probability of detection rather than the level of punishments. In Polinsky and Rubinfeld (1991) it was found that it may be optimal (for some parameter values) to punish repeat offenders more severely, when the government cannot observe illicit gains from criminal activities.

In Burnovski and Safra (1994) agents decide on the optimal number of crimes. They show that reducing the sanctions on subsequent crimes while increasing the penalty on previous crimes will reduce the overall criminal activity, if the probability of detection is sufficiently small. Our analysis is very similar to their paper. They also consider an n-periods framework. The main difference is that they search for the most deterring sequential policy for repeat offenders without taking into account that the regulator also has an objective to minimize the enforcement costs.

In Polinsky and Shavell (1998) agents live for two periods and can commit a crime twice. Their result is that young first-time offenders and old-second time offenders are penalized with the maximum sanction. Dana (2001) argues that, contrary to what is frequently assumed in the literature, probabilities of detection increase for repeat offenders. As a result, the optimal deterrence model a la Becker dictates declining, rather than escalating, penalties for repeat offenders.

Finally, the paper by Emons (2003) studies a two-period model, where agents may commit a crime two times. One important assumption of his model is that the agents are wealth constrained so that increasing the fine for the first offence means a reduction in the possible sanction for subsequent offences and vice versa. He also assumes that, besides crime deterrence, the main objective of the regulator is minimization of enforcement costs. The paper concludes that the optimal penalty structure should be declining in the number of offenses.

To summarize the results of earlier papers we conclude that the main argument in favor of decreasing penalty schemes is that probabilities of detection, usually, increase for repeat offenders and then Becker's model implies declining sanctions. The main policy implication from this analysis would be that sanctions should be declining when the regulator is resource constrained and offenders are wealth constrained. On the other hand, it is intuitively clear that recidivistic behavior should be punished more severely than first time offences or crimes committed by accident, since it usually signals a more grave criminal intent. With respect to policy implications, this scheme should be applied when the government cannot observe illicit gains from criminal activities[1].

In this paper we first analyze an n-period repeated game, where the regulator's main objective is to block any violations of law and, at the same time, to minimize the costs of crime control. We describe a forward looking solution, i.e. the regulator can commit to a certain policy from the beginning of the game and does not change the parameters of the penalty scheme (fine and probability of control) till the end of the planning horizon. The solution of this problem gives the desired result of complete deterrence. Even the first crime never happens, unless benefits from crime are much higher than the initial wealth of the offender. In the model of section 2 we rule out this possibility. The main intuition that drives this result comes from the fact that the agent pays the sanction for the first offence with probability p (rate of law enforcement). While any further sanction will be paid with lower probability, since the second offence can be detected only conditional on the fact that the first offence has been discovered. Hence, since paying the first fine is more likely than paying any subsequent fine, shifting resources from the last periods to the first increases deterrence for given rate of law enforcement. Consequently, as in Emons (2003), p is minimized by putting all scarce resources into the penalty for the first

[1]See Polinsky and Rubinfeld (1991).

detected offence.

However, the outcome will be different in case the regulator follows a time consistent (subgame perfect) strategy. This implies that the government is able to change its policy every period, conditioning its choice on the outcome of the preceding periods. In this case the regulator chooses the optimal subgame perfect action in the beginning of each period. In section 3 we show that the scheme derived as a forward looking solution in case of full commitment is not a time consistent (subgame perfect) strategy for the regulator in a multi-period setting. Section 4 concludes.

2 Multi-period Model, Forward Looking Solution (Full Commitment Case)

We consider a multi-period optimization problem of a cost minimizing regulator (antitrust authority or police) whose aim it is to block violations of law (for example, violations of antitrust law, violations of criminal law, violations of pollution standards).

We consider a continuum of potential offenders which has measure 1. Individuals or firms live for n periods. In each period the agents can engage in an illegal activity, such as polluting the environment, evading taxes, or violating competition law. If an agent commits the act in either period he receives a monetary benefit $b > 0$. Following Polinsky and Rubinfeld (1991) b is the illicit gain and the crime creates no acceptable gain. The act causes a monetary harm $h > 0$ to society and, thus, has to be deterred. We assume that the following inequality is satisfied, $h > b$. So, the act is not socially desirable.

To achieve deterrence the government chooses sanctions and a probability of apprehension. The regulator cannot tell in which period of its life the individual is. It can only observe the information after the crime has been discovered. Hence, the regulator only observes whether the crime is the first or second or n^{th} one. Accordingly, the government applies fines $s_1, s_2, ..., s_n \geq 0$, where s_i is the penalty in case the offense by this particular agent is recorded by the authority already i times. Moreover, the government chooses a rate of law enforcement, p, which can also be seen as the probability of conviction. We assume that p is the same for all (first time and repeated) offenses. Since apprehension is costly, the government wishes to minimize p and reduce the number of crimes. The overall objective of the regulator is to minimize the number of crimes. Subject to that objective being reached, the regulator aims to minimize costs of control, p. So, the objective function of the regulator can be written as $max - (p + Hk)$, where p is the probability of control (or rate of law enforcement), k is the number of crimes, and H is the disutility from crime for the regulator, which is assumed to be a large positive number.

The agents are risk neutral and maximize expected income. They have initial wealth W and hold it over all n periods unless the government interferes with sanctions. Benefits from crime b are consumed immediately, and the maximum of what the government can extract from the agents is W.[2] Moreover, based on Becker's (1968) maximum fine result, we assume that in order to minimize p the government will use the agent's entire wealth for sanctions. This implies that the fines $s_1, s_2, ..., s_n$ have to satisfy the "budget constraint" $\sum_{i=1}^{n} s_i = W$. To simplify the analysis we also assume no discounting.

An agent chooses the number of crime that can be committed or, in other words, he (she) can choose between following strategies:

Not to commit a criminal act (i.e. not to participate in a cartel) at all. Then the utility from this strategy for the "offender" has the following form $U(0, 0, ..., 0) = W$.

Commit crime (collude) only once in any of the periods.

The utility from this strategy for the offender equals $U(1, 0, ..., 0) = W + b - ps_1$.

Commit crime in any two periods: $U(1, 1, 0, ..., 0) = W + b - ps_1 + b - p(1-p)s_1 - p^2 s_2$.

Commit crime in any three periods: $U(1, 1, 1, ..., 0) = U(0, 1, 1, 1, ..., 0) = U(0, 0, ..., 0, 1, 1, 1) = W + b - ps_1 + b - p(1-p)s_1 - p^2 s_2 + b - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3$.

....

Commit crime in all n periods:

$U(1, 1, 1, ..., 1) = W + b - ps_1 + b - p(1-p)s_1 - p^2 s_2 + b - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3 + ... + b - (C^0_{n-1}(1-p)^{n-1} ps_1 + C^1_{n-1}(1-p)^{n-2} p^2 s_2 + C^2_{n-1}(1-p)^{n-3} p^3 s_3 + ... +$

[2] This assumption seems to be not quite realistic. In most of the cases, for example in case of tax evasion or illegal price-fixing activities, the penalty takes in to account not only initial wealth of the firm but also accumulated rents from illegal activities. However, this assumption is adopted here to focus on obtaining analytical results with respect to establishing an optimal sequence of sanctions.

$C_{n-1}^{n-2}(1-p)p^{n-1}s_{n-1} + C_{n-1}^{n-1}p^n s_n)$,

where coefficients of these polynomials are formed according to the following formula:

$$C_h^k = \frac{h!}{k!(h-k)!}, \quad h \geq k.$$

To clarify the notation:

$b - ps_1$ is the expected benefit from the first detected crime

$b - p(1-p)s_1 - p^2 s_2$ is the expected benefit from the second detected crime

$b - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3$ is the expected benefit from the third detected crime

$b - (C_{n-1}^0(1-p)^{n-1}ps_1 + C_{n-1}^1(1-p)^{n-2}p^2 s_2 + C_{n-1}^2(1-p)^{n-3}p^3 s_3 + ... + C_{n-1}^{n-2}(1-p)p^{n-1}s_{n-1} + C_{n-1}^{n-1}p^n s_n)$ is the expected benefit from n^{th} detected crime.

We impose the following assumptions on the parameters $0 < p < 1, b > 0, W > 0$. The possibility $p = 0$ does not make sense, since then there is no threat for the agent to be convicted and no way to prove the criminal to be guilty.

We also assume here that agents have enough wealth so that deterrence is always possible, i.e., $nb < \sum_{i=1}^{n} s_i \leq W$. Further, we derive sanctions that give the agents the proper incentives not to engage in criminal activities in either period. This means, we derive a penalty scheme which ensures $U(1,0,...,0) < U(0,0,...,0)$, $U(1,1,...,0) < U(0,0,...,0)$,..., $U(1,1,...,1) < U(0,0,...,0)$. These are included as constraints in the optimization model. The main objective of the regulator is crime prevention and minimization of costs of law enforcement, i.e. minimization of p. This leads to the following model.

The aim of the regulator to prevent crime and to minimize the enforcement costs is reflected in the objective function (1) below, while the aim to provide incentives for the agents not to commit any crime is reflected in incentive constraints (2)-(n+1).

$$min \;\; p + Hk \tag{1}$$

s.t.

$$b - ps_1 \leqq 0 \tag{2}$$

$$2b - ps_1 - p(1-p)s_1 - p^2 s_2 \leqq 0 \tag{3}$$

$$3b - ps_1 - p(1-p)s_1 - p^2 s_2 - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3 \leqq 0 \tag{4}$$

....

$$lb - \sum_{h=1}^{l}\sum_{k=1}^{h} C_{h-1}^{k-1}(1-p)^{h-k}p^k s_k \leqq 0 \tag{l+1}$$

....

$$nb - \sum_{h=1}^{n}\sum_{k=1}^{h} C_{h-1}^{k-1}(1-p)^{h-k}p^k s_k \leqq 0 \tag{n+1}$$

$$s_1 + s_2 + ... + s_{n-1} \leqq W \tag{n+2}$$

$$s_1 \geq 0, s_2 \geq 0, ..., s_{n-1} \geq 0, p > 0. \tag{n+3}$$

The Lagrangian for this problem has the following form:

$$L = -p - \sum_{j=1}^{n} \lambda_j [jb - \sum_{h=1}^{j}\sum_{k=1}^{h} C_{h-1}^{k-1}(1-p)^{h-k}p^k s_k] - \lambda^*(\ s_1 + s_2 + ... + s_{n-1} - w) \tag{5}$$

Using Kuhn-Tucker conditions to solve the minimization problem (1)-(n+3), we obtain the result stated in Proposition 1.

Proposition 1 *The optimal cost minimizing sanction scheme sets the penalty for the first detected violation equal to the entire wealth of the agent and for all subsequent violations the penalties will be equal to zero, i.e.* $s_1^* = W$ *and* $s_2^* = ... = s_n^* = 0$. *The probability of law enforcement is constant over time and equals* p^*, *which represents the smallest positive solution of the polynomial of order* n *in* p, *given by expression (19).*

The proof of Proposition 1 consists of several steps: first, we derive FOCs and complementary slackness conditions of the minimization problem described above; second, based on the FOCs we prove Lemma 2, which states that inequality $\frac{\partial L}{\partial s_l} > \frac{\partial L}{\partial s_{l+1}}$ holds for any time period $l \in \{1, ..., n-1\}$; finally, applying Lemma 2 and the complementary slackness conditions we obtain the optimal penalty schedule with $s_1^* = W$ and $s_2^* = ... = s_n^* = 0$ and $p > 0$.

Proof. To derive the FOCs we take partial derivatives of expression (5) with respect to all $n-1$ variables, which denote the penalties in the corresponding periods. Recall that, taking into account that the budget constraint must be binding, s_n can be expressed through all the unknowns and initial wealth as follows $s_n = W - \sum_{i=1}^{n-1} s_i$. So, differentiating and simplifying the expressions, we obtain $n-1$ FOCs with respect to penalties in corresponding periods (6)-(10) and one FOC with respect to the probability of law enforcement (15). We also write down $n+1$ complementary-slackness conditions in expressions (11)-(14) below.

$$\frac{\partial L}{\partial s_1} = p(1-p)^0 \sum_{i=k+1}^{n} \lambda_i + \sum_{k=1}^{n-1} [C_k^0 p(1-p)^k (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \; if \; s_1 > 0) \tag{6}$$

$$\frac{\partial L}{\partial s_2} = \sum_{k=1}^{n-1} [C_k^1 p^2 (1-p)^{k-1} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \; if \; s_2 > 0) \tag{7}$$

$$\frac{\partial L}{\partial s_3} = \sum_{k=2}^{n-1} [C_k^2 p^3 (1-p)^{k-2} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \; if \; s_3 > 0) \tag{8}$$

....

$$\frac{\partial L}{\partial s_l} = \sum_{k=l-1}^{n-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \; if \; s_l > 0) \tag{9}$$

....

$$\frac{\partial L}{\partial s_{n-1}} = \sum_{k=n-2}^{n-1} [C_k^{n-2} p^{n-1} (1-p)^{k-(n-2)} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n - \lambda^* \leq 0 \quad (= 0 \; if \; s_{n-1} > 0) \tag{10}$$

Complementary slackness conditions are:

$$\lambda_1 \geq 0 \quad (\quad \lambda_1 * (2) = 0 \quad) \tag{11}$$

$$\lambda_2 \geq 0 \quad (\quad \lambda_2 * (3) = 0 \quad) \tag{12}$$

....

$$\lambda_n \geq 0 \quad (\quad \lambda_n * (n+1) = 0 \quad) \tag{13}$$

$$\lambda^* \geq 0 \quad (\quad \lambda^* * (\sum_{i=1}^{n-1} s_i - W) = 0 \;) \tag{14}$$

$$\frac{\partial L}{\partial p} = 0. \tag{15}$$

Next, we prove the following lemma.

Lemma 2 *For any $l \in \{1, ..., n-1\}$, $\frac{\partial L}{\partial s_l} > \frac{\partial L}{\partial s_{l+1}}$.*

Proof. The proof of this lemma is based on mathematical induction.

1. First, we show that the result stated in Lemma 2 holds in case the number of periods equals to three, i.e. $n = 3$.

We take $n = 3$, which implies that $k \in \{1, 2, 3\}$ and $l \in \{1, 2\}$.

Consequently, for $l = 1$, we obtain from (6) and (7) that

$$\frac{\partial L}{\partial s_1} - \frac{\partial L}{\partial s_2} = p(1-p)^0 \sum_{i=1}^{3} \lambda_i + \sum_{k=1}^{2} [C_k^0 p(1-p)^k (\sum_{i=k+1}^{n} \lambda_i)] - \sum_{k=1}^{2} [C_k^1 p^2 (1-p)^{k-1} (\sum_{i=k+1}^{3} \lambda_i)] =$$

$$= p\lambda_1 + 2p(1-p)\lambda_2 + (p^2 - 2p + 1)\lambda_3 > 0$$

Similarly, for $l = 2$, we obtain from (7) and (8) that

$$\frac{\partial L}{\partial s_2} - \frac{\partial L}{\partial s_3} = \sum_{k=1}^{2} [C_k^1 p^2 (1-p)^{k-1} (\sum_{i=k+1}^{3} \lambda_i)] - \sum_{k=2}^{2} [C_k^2 p^3 (1-p)^{k-2} (\sum_{i=k+1}^{3} \lambda_i)] =$$

$$= C_1^1 p^2 (\sum_{i=2}^{3} \lambda_i) + C_2^1 p^2 (1-p)^1 \lambda_3 - C_2^2 p^3 \lambda_3 = p^2 (\sum_{i=2}^{3} \lambda_i) + 2p^2 \lambda_3 - 3p^3 \lambda_3 = p^2 \lambda_2 + 3p^2 (1-p) \lambda_3 > 0.$$

2. Next, we show that the result stated in Lemma 2 holds for any arbitrary number of periods. For that purpose we show that if the result of Lemma 2 holds for $n = m$, then it also holds for $n = m + 1$.

Now, assume that

$$\frac{\partial L}{\partial s_l} - \frac{\partial L}{\partial s_{l+1}} = \sum_{k=l-1}^{m-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m} \lambda_i)] - \sum_{k=l}^{m-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m} \lambda_i)] > 0 \tag{16}$$

is true for any $1 < l \leq n$ when $n = m$.

Based on this we have to prove that

$$\sum_{k=l-1}^{n-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{n} \lambda_i)] - \sum_{k=l}^{n-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{n} \lambda_i)] > 0$$

is true for any $1 < l \leq n$ when $n = m + 1$. Clearly,

$$\sum_{k=l-1}^{m} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m+1} \lambda_i)] - \sum_{k=l}^{m} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m+1} \lambda_i)] =$$

$$= \sum_{k=l-1}^{m-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m+1} \lambda_i)] - \sum_{k=l}^{m-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m+1} \lambda_i)] + C_m^{l-1} p^l (1-p)^{m-(l-1)} \lambda_{m+1}) -$$

$$- C_m^l p^{l+1} (1-p)^{m-l} \lambda_{m+1} =$$

$$= \sum_{k=l-1}^{m-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m} \lambda_i)] - \sum_{k=l}^{m-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m} \lambda_i)] + \lambda_{m+1} \sum_{k=l-1}^{m-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)}] -$$

$$- \lambda_{m+1} \sum_{k=l}^{m-1} [C_k^l p^{l+1} (1-p)^{k-l}] + C_m^{l-1} p^l (1-p)^{m-(l-1)} \lambda_{m+1} - C_m^l p^{l+1} (1-p)^{m-l} \lambda_{m+1} =$$

$$= \sum_{k=l-1}^{m-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{m} \lambda_i)] - \sum_{k=l}^{m-1} [C_k^l p^{l+1} (1-p)^{k-l} (\sum_{i=k+1}^{m} \lambda_i)] + \tag{17}$$

$$+ \lambda_{m+1} \left\{ \sum_{k=l-1}^{m} [C_k^{l-1} p^l (1-p)^{k-(l-1)}] - \sum_{k=l}^{m} [C_k^l p^{l+1} (1-p)^{k-l}] \right\} > 0. \tag{18}$$

Expression (17) is positive due to (16), while (18) will be strictly positive for any $p < \frac{1}{2}$, which corresponds to current rates of law enforcement for major types of economic crimes. ■

Next, using the result of Lemma 2, we derive the optimal penalty schedule.

We start by showing that it is impossible that constraint (n+2) is not binding.

In case this constraint is not binding, there are three possibilities:

1. $\sum_{i=1}^{n-1} s_i < W$ and $s_i > 0$ for all $i \in \{1, ..., n-1\}$,
2. $\sum_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for all $i \in \{1, ..., n-1\}$,
3. $\sum_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for some $i \in \{1, ..., n-1\}$.

The result of Lemma 2 immediately implies that the solution with $s_i > 0$ for all $i \in \{1, ..., n-1\}$ is impossible.

Consider $\sum_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for all $i \in \{1, ..., n-1\}$. Then the first order conditions (6)-(10) imply that $\frac{\partial L}{\partial s_1} < 0, \frac{\partial L}{\partial s_2} < 0, ..., \frac{\partial L}{\partial s_{n-1}} < 0$. Moreover, it holds that $\lambda^* = 0$. This implies that (9) becomes

$$\frac{\partial L}{\partial s_l} = \sum_{k=l-1}^{n-1} [C_k^{l-1} p^l (1-p)^{k-(l-1)} (\sum_{i=k+1}^{n} \lambda_i)] - \lambda_n p^n < 0 \quad \text{for all } \ l \in \{1, ..., n-1\}.$$

However, take the last period $l = n-1$, then

$$\frac{\partial L}{\partial s_{n-1}} = \sum_{k=n-2}^{n-1}[C_k^{n-2}p^{n-1}(1-p)^{k-(n-2)}(\sum_{i=k+1}^{n}\lambda_i)] - \lambda_n p^n = p^{n-1}\lambda_{n-1} + np^{n-1}\lambda_n(1-p) > 0.$$

Hence, condition (10) cannot be strictly negative. This implies that the outcome with $s_i = 0$ for all $i \in [1, n-1]$ and $\lambda^* = 0$ cannot arise as a solution of the minimization problem of the regulator.

Next, consider $\sum\limits_{i=1}^{n-1} s_i < W$ and $s_i = 0$ for some $i \in \{1, ..., n-1\}$. Assume $s_l = 0$ for $l < n-1$. This means that (9) must be non-positive, i.e.

$$\frac{\partial L}{\partial s_l} = \sum_{k=l-1}^{n-1}[C_k^{l-1}p^l(1-p)^{k-(l-1)}(\sum_{i=k+1}^{n}\lambda_i)] - \lambda_n p^n < 0.$$

But we have just shown that

$$\frac{\partial L}{\partial s_{n-1}} = \sum_{k=n-2}^{n-1}[C_k^{n-2}p^{n-1}(1-p)^{k-(n-2)}(\sum_{i=k+1}^{n}\lambda_i)] - \lambda_n p^n > 0$$

and, hence, using Lemma 2, we can conclude that this outcome also cannot be a solution.

The outcome with $\sum\limits_{i=1}^{n-1} s_i = W$ and $s_i = 0$ for $i < k \in \{1, ..., n-1\}$ and $s_l > 0$ for $l > k \in \{1, ..., n-1\}$ is impossible due to the result of Lemma 2.

Moreover, the outcome with $\sum\limits_{i=1}^{n-1} s_i = W$ and $s_1 > 0$, $s_2 > 0$ and $s_i = 0$ for all $i \in \{3, ..., n-1\}$ cannot arise. Consider $s_1 > 0$, $s_2 > 0$. Using (6) and (7) we obtain that $\frac{\partial L}{\partial s_1} = \frac{\partial L}{\partial s_2} = 0$. But this contradicts the result of Lemma 2, which states that $\frac{\partial L}{\partial s_1} > \frac{\partial L}{\partial s_2}$.

We conclude that only the following is possible: $s_1^* > 0$, $s_2^* = ... = s_n^* = 0$ and $\sum\limits_{i=1}^{n-1} s_i = W$, which implies that $s_1^* = W$, $s_2^* = ... = s_n^* = 0$.

Finally, optimal behavior implies that only condition (n+1) on the benefits from crime will be binding, so that $\lambda_1 = \lambda_2 = ... = \lambda_{n-1} = 0$ and $\lambda_n \geq 0$. Hence, the expressions for the optimal probability of law enforcement, p^*, λ^*, and λ_n will be determined from condition (n+1), $\frac{\partial L}{\partial s_1} = 0$, and $\frac{\partial L}{\partial p} = 0$.

In particular, p^* is represented as a solution of the polynomial of order n (19) with $s_1 = W$, $s_2 = 0, ..., s_n = 0$.

$$nb - \sum_{h=1}^{n}\sum_{k=1}^{h} C_{h-1}^{k-1}(1-p)^{h-k}p^k s_k = 0 \tag{19}$$

Next we present the proof of the fact that only constraint (n+1) on the benefits from crime will be binding.

Proof. We can show that for the penalty scheme given by $s_1^* = W$, $s_2^* = ... = s_n^* = 0$, only condition $(n+1)$ can be binding and, hence, p^* is found as a solution of the polynomial of order n given in (19).

The main intuition for the proof of this result is the observation that only constraint $(n+1)$ can be binding due to the construction of the problem. Assume, for example, that constraint $(l+1)$ is binding for some $l \in \{2, ..., n-1\}$, then it follows that the LHS of the constraint $(l+2)$ has to be strictly positive, which is impossible by construction of the problem.

Now we prove this statement using rigorous mathematical tools. First, we consider the situation where constraint $(l+1)$ is binding. It can be written as follows:

$b - ps_1 + b - p(1-p)s_1 - p^2 s_2 + b - (1-p)^2 ps_1 - 2p^2(1-p)s_2 - p^3 s_3 + ... + b - (C_{l-1}^0(1-p)^{l-1}ps_1 + C_{l-1}^1(1-p)^{l-2}p^2 s_2 + ... + C_{l-1}^{l-1}p^n s_l) = 0.$

At the same time constraint $(l+2)$ can be written as follows

$$(l+1) + b - (C_l^0(1-p)^l ps_1 + C_l^1(1-p)^l p^2 s_2 + ... + C_l^l p^n s_{l+1}).$$

Now, taking into account that $s_1^* = W$, $s_2^* = ... = s_n^* = 0$ and $(l+1) = 0$, constraint $(l+2)$ can be rewritten as

$$b - C_l^0(1-p)^l pW = b - (1-p)^l pW. \tag{20}$$

Moreover, using the formula for finite geometric series and the fact that $s_1^* = W$, $s_2^* = ... = s_n^* = 0$, constraint $(l+1)$ can be rewritten as follows.

$$(l+1) = bl - pW(1 + (1-p) + (1-p)^2 + ... + (1-p)^{l-1}) = bl - W(1-(1-p)^l).$$

Recall that constrained $(l+1)$ is binding. This implies that $b = \frac{W(1-(1-p)^l)}{l}$. Now expression for constraint $(l+2)$ in (20) becomes

$$\frac{W(1-(1-p)^l)}{l} - (1-p)^l pW = \frac{W}{l}(1-(1-p)^l - pl(1-p)^l)$$

It is easy to show that the first derivative of this expression with respect to p is strictly positive for any $0 < p < 1$, $l > 0$, and $W > 0$. Hence, this function is strictly increasing in p for any $0 < p < 1$, $l > 0$, and $W > 0$. At the same time function $\frac{W}{l}(1 - (1-p)^l - pl(1-p)^l) = 0$ when $p = 0$. Hence, this expression is strictly positive for any $0 < p < 1$.

This proves the fact that given that constraint $(l+1)$ is binding, it must hold that the LHS of the constraint $(l+2)$ in the problem (1)-(n+1) must be strictly positive, but this would contradict the construction of optimization problem. ■

End of the proof of Proposition 1. ■

The main intuition behind this proposition is very simple. It follows immediately from any of the incentive constraints (3)-(n+1). The agent pays the sanction s_1 with probability p, while any further sanction will be paid with lower probability : s_2 with probability p^2, s_3 with probability p^3, and s_n only with probability p^n. In other words, the agent is charged s_2 with probability p only if he has paid already s_1 . Hence, since paying the first fine is more likely than paying any subsequent fine, shifting resources from the last periods to s_1 increases deterrence for given p. Consequently, as in Emons (2003), p is minimized by putting all scarce resources into s_1.

Example 3 *Figure 1 illustrates the proof graphically in the* (p, s_1)*-diagram for the two-period case. The game in this case is described as follows. A strategy of player 1 (regulator) is given by* $\sigma = (p, s_1, s_2)$*, while the strategy set of player 2 (offender) is given by* $\{0, 1, 2\}$.

In case $n = 2$*, the optimization problem of the regulator will be as follows:*

$$min \;\; p + Hk$$

s.t.

$$b - ps_1 \leqq 0 \qquad (1)$$

$$2b - ps_1 - p(1-p)s_1 - p^2 s_2 \leqq 0 \qquad (2)$$

$$s_1 + s_2 \leqq W \qquad (3)$$

$$0 < p \leqq 1.$$

Suppose $b \geqq 0$, $W > 2b$, $s_2 = W - s_1$.

Graphically, the solution of this problem, which has the form $s_1^* = W$, $s_2^* = 0$, $p = p^*$, *is represented in Figure 1, where the parameter values are* $b = 1$, $W = 3$.

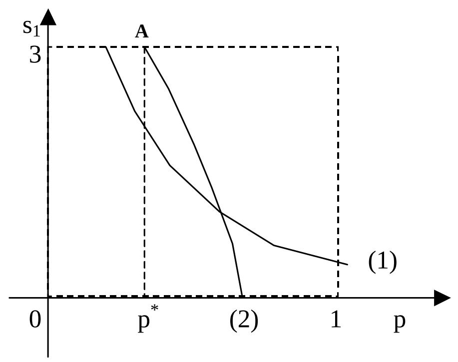

Figure 1: Graphical illustration of the solution in two-period case.

The solution of the problem is represented by point A in Figure 1, with $s_1 = W$, $s_1 = 0$, $p = p^* > 0$. *In general, for the n-period case this diagram will be an n-dimensional* $(p, s_1, ..., s_{n-1})$ *and the solution of the problem will be represented by the point of the n-dimensional cone which is the closest to the vertical axis and satisfies all the incentive constraints.*

3 Optimal Sanctions if Government cannot Commit

In this section we investigate under which conditions the sanction scheme described in Proposition 1 is sub-game perfect. This means: Does the government really implement these sanctions once the agent has committed a crime? To do so, we study the subgame starting when the agent has been apprehended for the first time.

In the setting, where the regulator can change its strategy once the crime has occurred, the scheme described in section 2 will no longer be optimal. To show this we consider the subgame starting when the agent has been apprehended for the first time. If the government sticks to the penalty scheme described in Proposition 1, the agent will commit the second offence for sure because it comes for free. At the same time, in this setting the government's payoff is much higher in case it does not try to prevent the next period crime, since in this way it saves on costs of control. So, clearly, an equilibrium with the authority playing $p = 0$ and the agent committing the crime will be chosen in each period after the first conviction and, hence, can emerge as an SPNE of the multi-period repeated game. This implies that the scheme of Proposition 1 does not appear to be a time consistent (subgame perfect) strategy for a government in an n-period setting. Moreover, an argument given below shows that, if the government cannot commit, equal rather than decreasing sanctions will be optimal.

Derivation of an SPNE in No-Commitment Case[3]

Let us consider a finitely repeated game where objective functions and participation constraints of the players have exactly the same form as in the model of section 2. However, here we assume that the regulator can change its strategy in any period of the game, hence, also once the crime has occurred. The game in this case will be described as follows. A strategy of player 1 (regulator) is given by $\sigma = (p_1, ..., p_n, s_1, ..., s_n)$, while a strategy of player 2 (offender) is given by $k \in \{0, 1, 2, ..., n\}$. We also assume that agents have enough wealth so that deterrence is always possible, i.e., $nb < \sum_{i=1}^{n} s_i \leq W$.

Here we aim to check whether the outcome with penalty scheme set by the regulator (p_i and s_i for all $i \in \{1, ..., n\}$) such that any strategy for the firm except of strategy $(0, 0, ..., 0)$ will be blocked can arise as an SPNE of this game. Solving this game backwards we get the following results.

Consider the optimal strategy for the antitrust authority in the last period. Irrespective of what had happen before, in the beginning of period n the anti-trust authority solves the following problem, where p_n is probability of control as before, H reflects disutility of crime for the regulator, and I is an indicator function that is equal to 1 in case

[3]Unfortunately, we were only able to find one possible SPNE penalty scheme for which it holds that zero-crime outcome is sustained in equilibrium. However, we believe there exist many more SPNE penalty schemes, where some of them could have positive levels of crime in equilibrium as well. Hence, we could not characterize in general the set of SPNEs of the game in question. That is why, we state that the "equal" sanctions scheme is only an example of a possible policy that can reach full compliance behavior (k=0) in case the government cannot commit.

crime occurs in period n and 0 in case crime is blocked,

$$min \;\; p_n + H * I \tag{21}$$

s.t.

$$b - p_n s_n \leqq 0 \tag{22}$$

$$s_1 + ... + s_n \leqq W \tag{23}$$

$$0 \leqq p_n \leqq 1. \tag{24}$$

This problem shows that the primary aim of the regulator in the beginning of period n is to block the n^{th} period crime and this has to be achieved at the lowest possible cost. So, at time n the regulator chooses s_n and p_n such that $I = 0$ is achieved in the period n, but also such that the wealth that is left to the offender after the penalty s_n is paid is enough to block crimes in all the preceding periods. The $I = 0$ outcome in period n is ensured if constraint (22) is satisfied. Moreover, constraints (23) and (24) on the parameters of the penalty scheme must also be satisfied.

This implies that a possible solution of this problem has the following form: $p_n = \frac{b}{s_n}$ and $s_n = W - \sum_{j=1}^{n-1} s_j$.

Looking for an SPNE, now given that we have blocked the crime in period n, we will try to find the optimal strategy for antitrust authority in the period $n-1$. Again the solution boils down to finding the optimum of the following problem:

$$min \;\; p_{n-1} + H * I \tag{25}$$

s.t.

$$b - p_{n-1} s_{n-1} \leqq 0 \tag{26}$$

$$s_1 + ... + s_n \leqq W \tag{27}$$

$$0 \leqq p_{n-1} \leqq 1. \tag{28}$$

Which is also given by $p_{n-1} = \frac{b}{s_{n-1}}$ and $s_{n-1} = W - \sum_{j \neq n-1} s_j$.

The same solution we get for every period. Hence, in the beginning of the first period antitrust authority again solves a similar problem:

$$min \;\; p_1 + H * I \tag{29}$$

s.t.

$$b - p_1 s_1 \leqq 0 \tag{30}$$

$$s_1 + ... + s_n \leqq W \tag{31}$$

$$0 \leqq p_1 \leqq 1. \tag{32}$$

A solution is $p_1 = \frac{b}{s_1}$ and $s_1 = W - \sum_{j=2}^{n} s_j$.

Consequently, a possible SPNE strategy of the regulator that satisfies conditions $p_i = \frac{b}{s_i}$ and $s_i = W - \sum_{j \neq i} s_j$ is given in expression (33).

$$s_i = \frac{W}{n} \quad \text{and} \quad p_i = \frac{bn}{W} \quad \text{for all} \quad i \in \{1, ..., n\} \tag{33}$$

In this SPNE the firm chooses not to commit any offence in any of the periods and the regulator sets penalty and rate of law enforcement that are uniform over time.

The only condition for existence of this solution is $bn < W$, which is also respected in the model of section 2.

The above analysis implies that in the repeated game setting the optimal penalty scheme, which is the part of SPNE strategy, can be given by $s_i = \frac{W}{n}$ and $p_i = \frac{bn}{W}$ for all $i \in \{1, ...n\}$. In this SPNE of the repeated game both penalties and rate of law enforcement are uniform over time.

4 Conclusions

The main conclusion of this paper is the result that, when offenders are wealth constrained and the government is resource constrained and can commit to a certain policy throughout the whole planning horizon, cost minimizing deterrence is decreasing, rather than increasing, in the number of offenses. We prove that for the agents who may commit an act several times, optimal sanctions are such that the fine for the first crime equals the offender's entire wealth, and the fines are zero for all the subsequent crimes. Since the agent can only be a repeat offender if he has been a first-time offender, there are no further offenses if we completely deter the first one. This conclusion completely supports the result obtained by Emons (2003) for a two-period model.

This result contradicts the widely prevailing escalating penalties imbedded in many penal codes and sentencing guidelines. This puzzle still requires deep investigation in the law and economics literature. However, we should be careful to make too strong conclusions and policy implications on the basis of the model of Section 2, since, unfortunately, analogous to Emons (2004), this scheme does not appear to be a time consistent (subgame perfect) strategy for the government in an n-periods setting.

Finally, we suggest some extensions of the model described above. Introduction of history dependent strategies will make the analysis more complete but at the moment it does not seem to be analytically solvable. However, it seems that the main result, namely a declining penalty scheme, will arise as a solution of optimization problem in that case as well. Another possibility is to introduce the opportunity for both players to react to the actions of the rival. This suggests to extend this model to a repeated n-period game between the regulator and the offender. This also allows to consider the case when full commitment is not possible and the set of strategies for the firm will automatically include all history dependent and history independent strategies. A third extension would be to introduce discounting. But this will only increase incentives for the cost minimizing regulator to extract the fine as soon as possible, so that the arguments in favor of a declining penalty scheme will be even stronger.

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